Integrali "interessanti"

$$\int \mathbf{sen^n x \cdot cos^m x \cdot dx} = \frac{\mathbf{sen^{n+1}x \cdot cos^{m-1}x}}{\mathbf{n+m}} + \frac{\mathbf{m-1}}{\mathbf{n+m}} \int \mathbf{sen^n x \cdot cos^{m-2} x \cdot dx} =$$

$$= -\frac{\mathbf{sen^{n-1}x \cdot cos^{m+1}x}}{\mathbf{n+m}} + \frac{\mathbf{n-1}}{\mathbf{n+m}} \int \mathbf{sen^{n-2} x \cdot cos^{m} x \cdot dx}$$

Se è $(n=1 \wedge m=0)$ o viceversa si sceglie la formula con gli esponenti non negativi (è vero per n, m=1 , considerando che l'integrale a secondo membro moltiplicato 0 è comunque zero).

Inoltre se $n,m \in \mathbf{N}$ e $m \geq 2$ (con la notazione che "s" sta "per sen x" e "c" per "cos x") :

$$\int s^n c^m \cdot dx = \int (s^n c) \cdot c^{m-1} \cdot dx = \frac{s^{n+1}c^{m-1}}{n+1} - \frac{m-1}{n+1}\int s^{n+1}c^{m-2}(-s)\cdot dx = \frac{s^{n+1}c^{m-1}}{n+1} + \frac{m-1}{n+1}\int s^n\left(1-c^2\right)c^{m-2}\cdot dx =$$

$$= \frac{s^{n+1}c^{m-1}}{n+1} + \frac{m-1}{n+1}\int s^n c^{m-2}\cdot dx = \frac{s^{n+1}c^{m-1}}{n+1} + \frac{m-1}{n+1}\int s^n c^{m-2}\cdot dx - \frac{m-1}{n+1}\int s^n c^m \cdot dx \Rightarrow$$

$$\left(1+\frac{m-1}{n+1}\right)\int s^n c^m \cdot dx = \frac{s^{n+1}c^{m-1}}{n+1} + \frac{m-1}{n+1}\int s^n c^{m-2}\cdot dx \;\Rightarrow \int \mathbf{s^n c^m} \cdot \mathbf{dx} = \frac{\mathbf{s^{n+1}c^{m-1}}}{\mathbf{n+m}} + \frac{\mathbf{m-1}}{\mathbf{n+m}}\int \mathbf{s^n c^{m-2}} \cdot \mathbf{dx}\;;$$

Analogamente $\int s^n c^m \cdot dx = -\int s^{n-1}c^m(-s)\cdot dx = -\frac{s^{n-1}c^{m+1}}{m+1} + \frac{n-1}{m+1}\int s^{n-2}c^m\left(1-s^2\right)\cdot dx =$

$$= -\frac{s^{n-1}c^{m+1}}{m+1} + \frac{n-1}{m+1}\int s^{n-2}c^m \cdot dx - \frac{n-1}{m+1}\int s^n c^m \cdot dx \Rightarrow$$

$$\left(1+\frac{n-1}{m+1}\right)\int s^n c^m \cdot dx = -\frac{s^{n-1}c^{m+1}}{m+1} + \frac{n-1}{m+1}\int s^{n-2}c^m \cdot dx \;\Rightarrow \int \mathbf{s^n c^m} \cdot \mathbf{dx} = -\frac{\mathbf{s^{n-1}c^{m+1}}}{\mathbf{n+m}} + \frac{\mathbf{n-1}}{\mathbf{n+m}}\int \mathbf{s^{n-2} c^{m}} \cdot \mathbf{dx}\;.$$

ALTRI

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\cdot\sqrt{a^2-x^2} + a\cdot \text{arc sen}\frac{x}{a} + c_1\right) = \frac{1}{2}\left(x\cdot\sqrt{a^2-x^2} - a\cdot \text{arc cos}\frac{x}{a} + c_2\right), \quad \text{con } c_2 = c_1 + \pi.$$